Finite-temperature large acoustic polaron dynamics in quasi-one-dimensional molecular crystals

Zoran Ivić, Slobodan Zeković and Dragan Kostić
The "Vinča" Institute of Nuclear Sciences, Theoretical Physics Department, 020 11001 Belgrade, Serbia, Yugoslavia
(Received 17 June 2001; revised manuscript received 6 September 2001; published 25 January 2002)

We report the results of theoretical examinations of large polaron motion in one-dimensional (1D) molecular crystals under the influence of thermal fluctuations of the host lattice and constant electric field. Such a situation may arise in biological macromolecules such as an α helix where charge (electron) transfer may be achieved by a polaron (soliton) mechanism. In that case, the electric field represents the effective endogenous electric field which is always present in realistic conditions. We derive and solve the Fokker-Planck equation for the distribution function of the soliton's center-of-mass position. It is shown that the soliton effectively exhibits a random walk. Moreover, in order to examine statistical properties of the soliton wave function, we calculate the mean value of the soliton probability density: $\langle|\beta(x,t)|^2\rangle$ and we find that, for sufficiently large times, thermal fluctuations destruct the soliton, which transforms into the Gaussian packet. These results were used in order to estimate the relevance of the soliton model of charge transfer in polypeptide chains.

DOI: 10.1103/PhysRevE.65.021911 PACS number(s): 87.15.−v, 05.40.−a, 71.38.−k

There has been an extensive interest [1–14] in the study of large polaron (soliton) properties in the quasi-one-dimensional molecular chains in recent years. It arises due to the assumed key role of polarons and solitons in long-distance charge (electron, etc.) and intramolecular vibrational energy (amide-I quanta) transfer along the conducting polymers and organic salts, and biological macromolecules such as an α helix and DNA [1–4].

The idea of the possible relevance of the soliton mechanism in energy transfer in biological systems has caused a lot of controversy concerning both the theoretical foundation of the concept and the explanation of the experimental data within the framework of the so-called Davydov's model. The entire theory has been the subject of numerous critical reexaminations [7–12] and now it is evident that the original Davydov idea, i.e., the soliton creation on account of the *single* amide-I quantum self-trapping (ST), cannot explain intramolecular vibrational energy transfer in an α helix [7–11]. On the other hand, since the values of the physical parameters of an α helix satisfy conditions for the soliton creation and existence on the basis of an excess electron ST, the whole concept may be applicable to the electron transfer in these substances. However, an idealized theoretical model in which the only connection with the particular system are the values of the parameters appearing in the model Hamiltonian, must be improved in order to account for the influence of the perturbations which are always present in realistic conditions. This assumes the examination of the soliton dynamics and stability as modified due to the thermal fluctuations and various external fields.

This is the subject of the present paper, where we shall analyze the soliton dynamics under the influence of an effective electric field which, in principle, may simulate the influence of both endogenous and external electromagnetic (EM) fields affecting the charge migration in an α-helix molecule. The origins of these fields are very diverse and both DC and AC electric fields may appear in realistic systems. Thus, for example, they may arise as a consequence of the polar structure of the α helix and surrounding molecules, which, due to their thermal oscillations, should generate an EM field. Consequently, soliton created by an excess electron and local distortion of the α-helix chain is affected by the effective EM field coming of the α helix, surrounding molecules as well as various external fields. Here, we shall restrict ourselves to the examination of the impact of DC fields, however, our analysis may be relevant even for solitons driven by AC fields if the period of their oscillations exceeds a solitons lifetime. Such a situation may appear in the realistic systems since a solitons lifetime at biologically relevant temperatures is estimated to vary up to 10^{-10} s [8,14], while the bioelectromagnetic fields span the frequencies from the sub-Hertz region up to a few gigahertz [15,16]. If $\vec{E}_{\rm eff}$ denotes the vector of the effective electric field, then its influence on electron on the nth site of the molecular chain may be described in terms of scalar potential $V=-e\hat{\vec{r}}\cdot\vec{E}_{\rm eff}$ where $\hat{\vec{r}}=\vec{R}_0\Sigma_n nB_n^\dagger B_n$ denotes the position operator of the excess electron in the chain. Consequently, the model Hamiltonian, disregarding the term containing the electron energy on the nth site, irrelevant in the present context, now reads

$$H=-J\sum_n B_n^\dagger(B_{n+1}+B_{n-1})+\frac{1}{\sqrt{N}}\sum_{q,n} F_q e^{iqnR_0}B_n^\dagger B_n \times(a_q+a_{-q}^\dagger)+\sum_q \hbar\omega_q a_q^\dagger a_q-eER_0\sum_n nB_n^\dagger B_n. \quad (1)$$

Here, E denotes the component of the effective electric field directed along the α helix, while, as usual, the operators $B_n^+(B_n)$ describe the presence (absence) of the electron on nth peptide group (PG), a_q^+ and a_q are the acoustic phonon creations and annihilation operators. The meaning of the remaining parameters are: J, is the intersite dipole-dipole transfer integral, $\omega_q=\omega_B\sin|qR_0/2|$ is the phonon frequency ($\omega_B=2\sqrt{\kappa/M}$ is the phonon bandwidth, while κ and M denote the spring constant and mass of the PG, respectively), $F_q=2i\chi\sqrt{\hbar/2M\omega_q}\sin qR_0$ denotes the electron-phonon coupling parameter, χ is its strength, R_0 denotes lattice spacing, and finally, $N\gg 1$ is the number of the PG.

Since the system parameters for α helix satisfy the adiabaticity condition ($B\sim 2J/\hbar\omega_B \gg 1$), soliton dynamics may be described within the time-dependent variational method assuming the separability of the electron and phonon degrees of freedom (so called D_2 *ansatz*), which in the final instance, results with the known set of equations slightly modified due to the presence of the electric field

$$i\hbar\dot{\beta}(x,t)+JR_0^2\beta_{xx}(x,t)-\frac{1}{\sqrt{N}}\sum_q F_q e^{iqx}[\alpha_q(t)+\alpha^*_{-q}(t)]\beta(x,t)=-eEx\beta(x,t), \tag{2}$$

$$i\hbar\dot{\alpha}_q(t)=\hbar\omega_q\alpha_q(t)+\frac{1}{\sqrt{N}}\int_{-\infty}^{\infty}\frac{dx}{R_0}F_{-q}e^{-iqx}|\beta(x,t)|^2, \tag{3}$$

for the envelope wave function $\beta(x,t)$ and phonon coherent amplitudes $\alpha_q(t)$. The equation for phonon amplitudes may be easily integrated in the case of coherent motion of electron and surrounding lattice distortion, which assumes that $|\beta(x,t)|^2=|\beta(x-vt)|^2$. This implies the trivial time dependence ($\sim e^{-iqvt}$) of the second term on the right-hand side of Eq. (3), whose general solution reads, $\alpha_q(t)=\alpha_q(0)e^{-i\omega_q t}+\alpha_q^s(t)$, where the first term denotes the homogenous solution of Eq. (3) and corresponds to free phonons, while $\alpha_q^s(t)$ is the soliton part of the lattice distortion, given as

$$\alpha_q^s(t)=-\frac{1}{\sqrt{N}}\frac{F_{-q}}{\hbar(\omega_q-qv)}\int_{-\infty}^{\infty}\frac{dx}{R_0}e^{-iqx}|\beta(x,t)|^2 \equiv -\frac{1}{\sqrt{N}}\frac{F_{-q}e^{-iqvt}}{\hbar(\omega_q-qv)}\int_{-\infty}^{\infty}\frac{dz}{R_0}e^{-iqz}|\beta(z)|^2; \quad z=x-vt. \tag{4}$$

Substituting the above general solution of the equation of motion for phonon amplitudes into Eq. (2), we obtain the perturbed nonlinear Schrödinger equation (NSE)

$$i\hbar\dot{\beta}(x,t)+JR_0^2\beta_{xx}(x,t)+\frac{4E_B}{1-\left(\frac{v}{c}\right)^2}|\beta(x,t)|^2\beta(x,t)=-eEx\beta(x,t)+f(x,t)\beta(x,t). \tag{5}$$

Here, $E_B=1/N\Sigma_q|F_q|^2/\hbar\omega_q$ represents the small polaron binding energy. The only difference in respect to the usual procedure [3,6] is the accounting of the general solution for the phonon amplitudes consisting of the particular (so-called soliton-performed) solution and the homogeneous one, which represents the influence of the free phonons. The first term on the right-hand side of Eq. (5) comes from the effective electric field, while the term containing $f(x,t)$ represents the influence of the free phonons. The explicit form of $f(x,t)$ depends on the nature of the external perturbation and has the simple harmonic form $f(x,t)=A\sin(\omega t-kx)$ in the case of the soliton interaction with the monochromatic acoustic wave, or $f(x,t)=1/\sqrt{N}\Sigma_q F_q e^{iqx}\alpha_q(0)e^{-i\omega_q t}$ + c.c., in the case of interaction with the phonon wave packet. For the initial condition, corresponding to the lattice in thermal equilibrium $f(x,t)$ represents a random force determined by [13,14]

$$\langle f(x,t)\rangle=0,$$

$$\langle f(x,t)f(x',t')\rangle=k_BTE_BR_0\sum_{\pm}\delta[x-x'\pm c(t-t')]. \tag{6}$$

Here, $\langle\ldots\rangle$ denotes the averaging over the equilibrium phonon ensemble in the classical limit where the amplitudes α_q correspond to the classical limit of the phonon operators a_q.

In the absence of perturbation, Eq. (5) has the one-soliton solution $\beta(x,t)=\sqrt{\mu/2}e^{i(k_s x-\omega t)}\cosh^{-1}(\mu/R_0)(x-\xi)$, where $k_s=\hbar v/2JR_0^2$ represents soliton quasimomentum, $\mu=E_B/J(1-v^2/c^2)$ denotes the so-called soliton parameter having the meaning of its inverse width measured in units of lattice spacing. Finally, $\xi=x_0+vt$ denotes the soliton center-of-mass coordinate. The above solution describes the particlelike entity slowly propagating along the chain with the velocity (v) far less than the speed of sound $(c)(v\ll c)$ and carrying the energy $E_s=m^*v^2/2$ and momentum $P_s=m^*v$. Here, the soliton effective mass (m^*) may be expressed in terms of the electron-phonon coupling constant $S\sim E_B/\hbar\omega_B$ and electron effective band mass $m=\hbar^2/2JR_0^2$ as follows [10], $m^*=m(1+3\pi^2S^2/2)$. Due to the presence of perturbing random force, Eq. (5) is a nonintegrable one and soliton undergoes nontrivial dynamics manifested through the evolution of its parameters. In what follows, we shall consider the processes in the lowest order of the perturbation when the radiative decay of the soliton amplitude may be neglected and soliton dynamics are governed by the evolution of their momentum and center-of-mass coordinate. For that purpose, it is sufficient to consider the time evolution (balance) equation for the soliton (polaron) total momentum, which is the sum of the electron part and that of the lattice distortion accompanying it, $P_s=P_e+P_{\mathrm{def}}$, $P_{\mathrm{def}}=\hbar\Sigma_q q|\alpha_q^s(t)|^2$. Within the framework of the present paper, the balance equation for polaron momentum results with the following system of stochastic differential equations:

$$\dot{\xi}(t)=v; \quad \dot{v}=\frac{eE}{m^*}+\mathcal{F}(\xi,t), \tag{7}$$

where $\mathcal{F}(\xi,t)=1/m^*\int dx/R_0 f(x,t)\partial/\partial x|\beta(x,t)|^2$ represents an effective random "force." The above system follows directly from the set of evolution Eqs. (2) and (3) after the separation of the fluctuation part $[\alpha_q(0)e^{-i\omega_q t}]$ from the coherent (soliton) part of phonon amplitude. This result should be understood in the way that although the fluctuations couple directly only to the electronic part, they influence the polaron dynamics as whole. An analogous situation was considered by Davydov (Ref. [3]) who examined the case where external forces affected directly the phonon subsystem but the final result was the change of the total soliton momentum.

Up until now, no further approximations have been involved and the above Eqs are exact, provided that the adiabatic criterion is satisfied. Now we assume that the perturbation is weak so that one may take the usual solitonlike solution for $\beta(x,t)$ but the time dependence of the soliton parameters $v=v(t)$ and $\xi=\xi(t)$ must be accounted for in further calculations. Thus, substituting $|\beta(x,t)|^2 \equiv |\beta[x-\xi(t)]|^2$, we found the following correlators determining the statistical properties of the effective random field: $\langle \mathcal{F}(\xi,t)\rangle = 0$ and

$$\langle \mathcal{F}(\xi,t)\mathcal{F}(\xi',t')\rangle = \frac{k_B T E_B R_0}{m^{*2}c^2}\sum_{\pm}\int \frac{dx}{R_0}\int \frac{dx'}{R_0}|\beta(x,t)|^2|\beta(x',t')|^2 \times \frac{\partial^2}{\partial t\,\partial t'}\delta[x-x'\pm c(t-t')]. \tag{8}$$

Our basic assumption is that the soliton is slow ($c \gg v$) and that its shape is practically unaffected by the perturbations. That implies that $|\beta(x,t)|^2 \equiv |\beta[x-\xi(t)]|^2$ is strongly localized around the point $x=\xi(t)$, which justifies the following approximation:

$$\langle \mathcal{F}(\xi,t)\mathcal{F}(\xi',t')\rangle \approx \frac{k_B T E_B R_0}{m^{*2}c^3}\sum_{\pm}\int \frac{dx}{R_0}\int \frac{dx'}{R_0} \times |\beta[x-\xi(t)]|^2|\beta[x'-\xi(t')]|^2 \times \frac{\partial^2}{\partial t\,\partial t'}\delta\left[\frac{\xi(t)-\xi(t')}{c}\pm(t-t')\right]. \tag{9}$$

For slow solitons, the term $[\xi(t)-\xi(t')]/c \sim v/c(t-t')$ is much smaller then the quantity $t-t'$, and may be neglected. Thus, the above correlator may be successfully approximated with

$$\langle \mathcal{F}(\xi,t)\mathcal{F}(\xi',t')\rangle = \frac{k_B T E_B R_0}{m^{*2}c^3}\frac{\partial^2}{\partial t\ \partial t'}\delta(t-t'). \tag{10}$$

Equation (7) may be integrated once and attains the well-known form of the δ-correlated random process

$$\dot{\xi}(t) = v_0 + \frac{eEt}{m^*} + \mathcal{R}(t);$$

$$\langle \mathcal{R}(t)\mathcal{R}(t')\rangle = \frac{k_B T E_B R_0}{m^{*2}c^3}\delta(t-t'), \tag{11}$$

describing, in the absence of a driving field, the known phenomenon: particle diffusion in the field of random velocities [17]. This problem was examined by Flytzanis, Ivic, and Malomed [13] who derived the Fokker-Planck equation (FPE) for the distribution function of the soliton position $\mathcal{P}(\xi,t)$

$$\frac{\partial \mathcal{P}(\xi,t)}{\partial t} = D\frac{\partial^2 \mathcal{P}(\xi,t)}{\partial \xi^2}, \tag{12}$$

where $D = k_B T E_B R_0/2m^{*2}c^3$ represents the diffusion coefficient. The solution of the above equation is well known [13,17] and given by

$$\mathcal{P}(\xi,t) = \frac{R_0}{\sqrt{4\pi Dt}}\exp\left(-\frac{\xi^2}{4Dt}\right). \tag{13}$$

According to the general theory of stochastic processes [17], the term coming out of the driving field modifies the FPE as follows:

$$\frac{\partial \mathcal{P}(\xi,t)}{\partial t} = -\left(v_0 + \frac{eEt}{m^*}\right)\frac{\partial \mathcal{P}(\xi,t)}{\partial \xi} + D\frac{\partial^2 \mathcal{P}(\xi,t)}{\partial \xi^2}. \tag{14}$$

It can be solved easily by virtue of the simple changes of variables: $\eta = \xi - \xi_0 - v_0 t - eE/2m^* t^2$ and $t=t'$, which brings it in the form of Eq. (12). Here, ξ_0 and v_0 represent the soliton center-of-mass initial position and initial velocity, respectively. Thus, using its solution and rewriting it in terms of original variables, we finally have

$$\mathcal{P}(\xi,t) = \frac{R_0}{\sqrt{4\pi Dt}}\exp\left(-\frac{\left[\xi - \xi_0 - v_0 t - \frac{eE}{2m^*}t^2\right]^2}{4Dt}\right). \tag{15}$$

With the help of the last equation, one can examine statistical properties of soliton. For that purpose, we shall follow the procedure proposed in [18,19] and we shall calculate the mean intensity of the wave $I(x,t) \equiv \langle |\beta(x,t)|^2\rangle = \int_{-\infty}^{\infty} d\xi/R_0\ \mathcal{P}(\xi,t)|\beta[x-\xi(t)]|^2$, soliton mean center-of-mass coordinate $\langle \xi\rangle = \int_{-\infty}^{\infty} d\xi/R_0\ \xi\ \mathcal{P}(\xi,t)$, and finally the mean soliton width $\Delta x = \sqrt{\sigma}$ where $\sigma = \langle x^2\rangle - \langle x\rangle^2$. Here, $\langle x^p\rangle = \int_{-\infty}^{\infty} dx/R_0 \int_{-\infty}^{\infty} d\xi/R_0 x^p |\beta[x-\xi(t)]|^2\ \mathcal{P}(\xi,t)$. In such a way after some manipulations, we arrive at

$$I(x,t) \equiv \langle |\beta(x,t)|^2\rangle = \frac{R_0}{\sqrt{4\pi Dt}}\int_{-\infty}^{\infty}\frac{d\xi}{R_0}\exp \times \left[-\frac{(\xi - \langle\xi\rangle)^2}{4Dt}\right]|\beta[x-\xi(t)]|^2, \tag{16}$$

$$\langle \xi(t)\rangle = \xi_0 + v_0 t + \frac{eEt^2}{2m^*}, \tag{17}$$

$$\Delta x = \sqrt{(\Delta x_0)^2 + 2Dt.} \tag{18}$$

Here, $\Delta x_0 \sim R_0/\mu$ denotes the soliton width at zero temperature. That is, the random lattice fluctuation and electric-field result in the mean value of the wave field, which attains the form of a Gaussian wave packet, whose center-of-mass position, in the mean, is that of classically charged particles affected by the constant electric field. The width of such a pulse increases, as can be seen from Eq. (18), while at the same time, its magnitude decreases. In order to estimate the

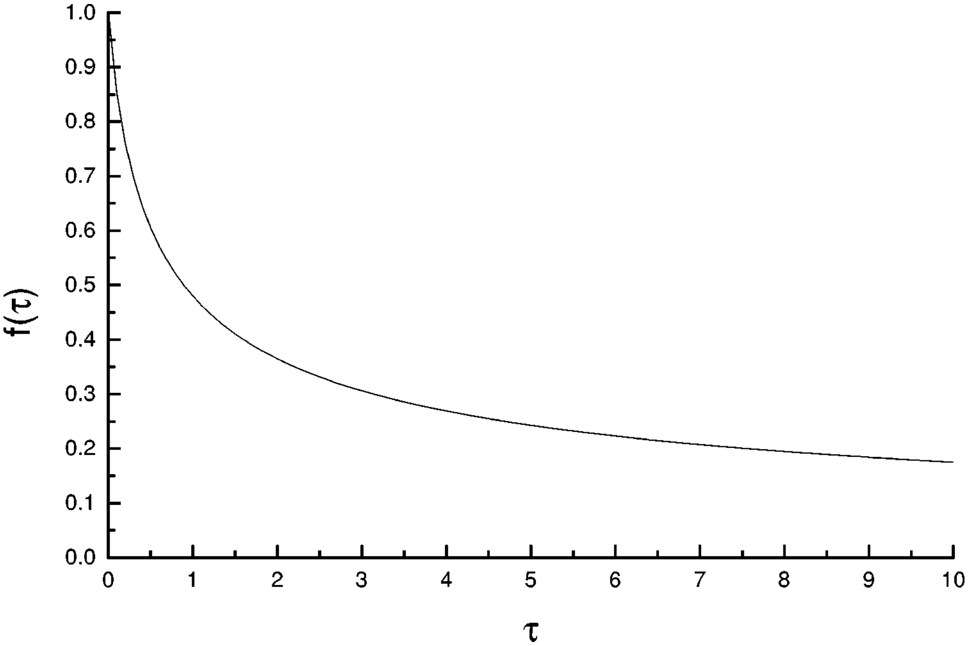

FIG. 1. Relative wave intensity $f(\tau)$ versus dimensionless time τ

rate of this damping, one should calculate $I(x,t)$, at least approximately. For that purpose, we substitute an explicit expression for the soliton envelope function in Eq. (16) and after some straightforward calculation, we obtain

$$I(x,t)=|\beta(x-\langle\xi\rangle)|^2\frac{R_0}{\sqrt{4\pi Dt}}\int_{-\infty}^{\infty}\frac{dy}{R_0}\times\frac{\exp\left(-\frac{y^2}{4Dt}\right)}{\cosh^2\frac{\mu y}{R_0}\left[1-\tanh\frac{\mu(x-\langle\xi\rangle)}{R_0}\tanh\frac{\mu y}{R_0}\right]^2} \tag{19}$$

At this stage, further calculations may be highly simplified if we transit into the moving reference frame in which $x=\langle\xi\rangle$ and where $|\beta[x=\langle\xi(t)\rangle]|^2\equiv\mu/2$, so that the wave intensity attains the simple form

$$I(x=\langle\xi\rangle,t)=\frac{\mu R_0}{2\sqrt{4\pi Dt}}\int_{-\infty}^{\infty}\frac{dy}{R_0}\frac{\exp\left(-\frac{y^2}{4Dt}\right)}{\cosh^2\frac{\mu y}{R_0}}. \tag{20}$$

It obviously decays in time, which is determined by the integral in the above expression. Unfortunately, the desired integral cannot be found exactly in the closed form. However, its numerical integration is relatively simple and in Fig. 1, we have plotted function $f(\tau)=2/\sqrt{\pi}\int_0^\infty dz\, e^{-z^2}\cosh^{-2}(\sqrt{4\tau}z)$, which represents relative wave intensity $I(x=\langle\xi\rangle,t)/I(x=\langle\xi\rangle,t=0)$. Here, τ represents dimensionless time measured in units of $R_0^2/D\mu^2$, i.e., $t=R_0^2/D\mu^2\tau$.

As one can see, the magnitude of the wave intensity rapidly decreases for short times ($\tau<1$) so that for $\tau\sim1$, it approaches the half of its initial value, so we have estimated its half lifetime as

$$t_{1/2}\sim\frac{(1+S^2)^2}{S^3T}\times10^{-11}\ \text{s}. \tag{21}$$

This last result does not depend on the electric field, which influences soliton translational motion only (E accelerates soliton), but does not affect its stability. This is the consequence of the exact solvability of NSE containing the driving term $\sim xE$ exclusively [i.e., if $f(x,t)=0$] which, with the help of a simple phase transformation [18], $\beta(x,t)=\phi(X,t)e^{ix\hbar\dot{\xi}/2JR_0^2}$, $X=x-\xi(t)$, and providing that $\xi(t)$ satisfies $\ddot{\xi}=eE/m^*$, may be reduced onto the standard form for envelope $\phi(X,t)$.

Our analysis shows that the soliton under the influence of the random lattice fluctuations and electric fields in, α helix undergoes Brownian motion, in which its center-of-mass position, on average, evolves in time similarly to the classical particle affected by the constant driving force. In addition, fluctuations result in the mean value of the wave field taking the form of a Gaussian wave packet whose width increases in time, while at the same time, its magnitude diminishes. The characteristic time scale of the dispersion and decay of this wave packet is given by the last equation, which points to the significant stability of soliton (large polaron) states in the electron-phonon systems provided that the large polaron existence criterion (adiabaticity-$B\gg1$ and weak coupling limit-$S\ll1$ [7,10,11]) is satisfied. In such a way, our results support the possible relevance of the solitonic mechanism in the charge (precisely, the electron-charge) transfer in biological macromolecules (α helix), since the demanded criterion is satisfied in these substances.

Let us now point out that in contrast to Ref. [14], where soliton radiation decay was considered, here we deal with the radiationless case. This concerns both soliton gradual decay into the delocalized (''band'') states and EM radiation, which may be generated due to its acceleration. The latter case was the subject of a recent study [20] where it was shown that the soliton motion along a polypeptide chain is affected by its periodic structure, which modulates soliton velocity and induces EM radiation. Comparing the present paper with results obtained in Ref. [14], it follows that soliton radiation decay takes place on the comparably larger time scale ($\tau_{rad} \sim (TS^5)^{-1} \times 10^{-10}$ s) and therefore is negligible in respect to the above-estimated one.

We would like to acknowledge useful conversations with Dr. D. Kapor. This work was supported by the Serbian Ministry of Science and Technology under Contract No. 01E15.

[1] A. J. Heeger, S. Kivelson, and J. R. Schrieffer, Rev. Mod. Phys. **60**, 781 (1988).
[2] E. G. Petrov, *Physics of Charge Transfer in Biological Systems* (Naukova Dumka, Kiev, 1984).
[3] A. S. Davydov, *Solitons in Molecular Systems* (Reidel, Dordrecht, 1985), Chap. 4.
[4] Chun-Min Chang, A. H. Castro Neto, and A. R. Bishop, e-print arXiv:cond-mat/0105249.
[5] A. H. Castro Neto and A. O. Caldeira, Phys. Rev. B **46**, 8858 (1992).
[6] A. S. Davydov and N. I. Kislukha, Phys. Status Solidi B **59**, 465 (1973).
[7] Z. Ivić and D. Brown, Phys. Rev. Lett. **63**, 426 (1989); D. Brown and Z. Ivić, Phys. Rev. B **40**, 9876 (1989).
[8] See, for example, *Davidov's Soliton Revisited*, edited by P. L. Christiansen and A. C. Scott (Plenum, New York, 1990).
[9] D. W. Brown, K. Lindenberg, and X. Wang, in *Davidov's Soliton Revisited* (Ref. [8].
[10] Z. Ivić, D. Kapor, M. Škrinjar, and Z. Popović, Phys. Rev. B **48**, 3721 (1993).
[11] Z. Ivić, Physica D **113**, 218 (1998).
[12] A. C. Scott, Phys. Rep. **217**, 1 (1992).
[13] N. Flytzanis, Z. Ivić, and B. A. Malomed, Europhys. Lett. **30**, 267 (1995).
[14] N. Flytzanis, Z. Ivić, and B. A. Malomed, J. Phys.: Condens. Matter **7**, 7843 (1995).
[15] C. W. Smith, Neural Networks **5**, 819 (1995).
[16] R. Höltzel, I. Lamprecht, Neural Networks **4**, 327 (1994).
[17] V. I. Klyatskin, *Stochastic Equations and Waves in Randomly Inhomogenius Media* (Nauka, Moscow, 1980) (in Russian).
[18] F. G. Bass, Yu. S. Kivshar, V. V. Konotop, and Yu. A. Sinitsyn, Phys. Rep. **157**, 63 (1988).
[19] I. M. Besieris, in *Nonlinear Electrodynamics*, edited by P. L. E. Uslengi, (Academic, New York, 1980).
[20] A. A. Eremko and L. S. Brizhik, in Proceedings of the International Symposium on Electromagnetic Aspects of Selforganization in Biology, Prague, Czech Republic, July 2000, (unpublished), p. 24.